MA651 Topology. Midterm Exam.

Due February 28, 2006

1. Let two positive real functions f_1 and f_2 on $[0,1]$ be equivalent if

$$0 < \liminf_{x \to 0} \frac{f_1(x)}{f_2(x)}, \ \limsup_{x \to 0} \frac{f_1(x)}{f_2(x)} < \infty$$

   Show that this is an equivalence relation, and that its quotient set is uncountable.

2. Let τ_1 and τ_2 be topologies on X. Show that the diagonal $\triangle = \{(x,x) : x \in X\}$ is closed in $(X,\tau_1) \times (X,\tau_2)$ if and only if for every $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $U_{x_1} \in \tau_1$ and $U_{x_2} \in \tau_2$, where $x_1 \in U_{x_1}$ and $x_2 \in U_{x_2}$ such that $U_{x_1} \cap U_{x_2} = \emptyset$.

3. Prove Theorem 31.5. (Let $\{Y_\alpha \mid \alpha \in \mathscr{A}\}$ be any family of spaces. $\prod_{\alpha \in \mathscr{A}} Y_\alpha$ is connected if and only if each Y_α is connected.)

4. Let X be a topological space, Y is an arbitrary set, and $f : X \to Y$ where f onto Y. The identification topology in Y is given by $\tau_f = \{U \subset Y : f^{-1}(U) \text{ is open in } X\}$. If X and Y are two topological spaces, then a continuous surjection $f : X \to Y$ is called an identification whenever the topology in Y is exactly the identification topology (that is the sets U open in Y if and only if $f^{-1}(U)$ is open in X). Prove that f is an identification if f is not only continuous surjection, but also open or closed map.

5. Let $X = \{a,b,c\}$, $\tau_X = \{\emptyset, X, \{a,b\}, \{b,c\}, \{b\}\}$, and let $Y = \{1,2,3\}$, $\tau_Y = \{\emptyset, Y, \{1\}, \{2\}, \{1,2\}\}$. Prove or disprove that $(X,\tau_X) \cong (Y,\tau_Y)$.